

6/MTH-350 Syllabus-2023

2026

(May-June)

FYUP : 6th Semester Examination

MATHEMATICS

(Complex Analysis)

(MTH-350)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions.

Answer **four** questions, taking **one** from each Unit.

UNIT—I

1. (a) If z and w be complex numbers, prove that

$$|z - w|^2 = |z|^2 - 2 \operatorname{Re} z \bar{w} + |w|^2 \quad 4$$

- (b) Find the real and imaginary parts of

$$\frac{3 + 5i}{7i + 1} \quad 4$$

(2)

(c) State and prove the triangle inequality of complex number. 5

(d) Calculate the square root of $2+3i$. 4

(e) Find the multiplicative inverse of
$$z = 5 + \frac{1}{5}i$$
 2

2. (a) Find the absolute value and conjugate of

$$\frac{3-i}{\sqrt{2}+3i}$$
 4

(b) Express $(1+i)^{-4}$ in the form $x+iy$. 4

(c) Find the equation of the circle through the points $1, -1, i$. 4

(d) Discuss the extended plane of complex numbers and its spherical representation. 5

(e) Show that
 $|z|^2 = z\bar{z}$ 2

(3)

UNIT—II

3. (a) State whether True or False with justification : 3

A convergent sequence is bounded.

(b) Let $\{a_n\}$ be a complex sequence and let

$$\limsup |a_n|^{1/n} = L$$

Then show that $\sum_{n=1}^{\infty} a_n$ converges absolutely if $L < 1$ and diverges if $L > 1$. 5

(c) Find the radius of convergence of

$$\sum_{n=1}^{\infty} n! z^n$$
 3

(d) Find the value of e^z for

$$z = -\frac{\pi i}{2} \text{ and } z = \frac{3}{4}\pi i$$
 4

(e) Discuss the sine function $\sin z$ for $z \in \mathbb{C}$ as a power series expansion about the origin. 4

(4)

4. (a) State whether True or False with justification : 3

The sequence

$$\left\{ \frac{\cos n + i \sin n}{n} \right\}$$

is convergent.

- (b) Define a Cauchy sequence of complex numbers. 2

- (c) Show that

$$\sum_{n=1}^{\infty} \left(\frac{1}{3^n} + i \frac{1}{4^n} \right) = \frac{1}{2} + \frac{i}{3} \quad 5$$

- (d) Show that the geometric series

$$\sum_{n=0}^{\infty} z^n$$

converges absolutely for $|z| < 1$. 3

- (e) Show that—

$$(i) \sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$$

$$(ii) \cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$$

3+3=6

(5)

UNIT—III

5. (a) Evaluate $\int_{\gamma} z^2 dz$, if

(i) γ is an arc of a circle;

(ii) γ is a closed circle. 3+3=6

- (b) If $f'(z) \equiv 0$ in a domain D , then show that $f(z)$ is constant. 5

- (c) Find the value of $\int \bar{z} dz$ along
 $z(t) = e^{2it}$, $-\pi \leq t \leq \pi$ 5

- (d) Show that
 $\frac{d}{dz}(e^z) = e^z$ 3

6. (a) Show that $f(z) = |z|^2 = x^2 + y^2$ is differentiable only at the origin. 4

- (b) Evaluate $\int_C |z|^2 dz$ along

$$(i) C = C_1; z_1(t) = t + it, 0 \leq t \leq 1$$

$$(ii) C = C_2; z_2(t) = t^2 + it, 0 \leq t \leq 1 \quad 6$$

(6)

- (c) Deduce Cauchy-Riemann equations for a complex-valued function

$$f(z) = u(x, y) + iv(x, y) \quad 5$$

- (d) If $f : G \rightarrow \mathbb{C}$ is differentiable at a point $a \in G$, then show that f is continuous at a .

4

UNIT—IV

7. (a) State and prove Liouville's theorem. $2+4=6$

- (b) Let f be analytic in $B(a; R)$ and suppose $|f(z)| \leq M$ for all $z \in B(a; R)$. Then show that

$$|f^{(n)}(a)| \leq \frac{n!M}{R^n} \quad 4$$

- (c) Evaluate

$$\int_{\gamma} \frac{\sin z}{z} dz$$

where $\gamma(t) = e^{it}$, $0 \leq t \leq 2\pi$. 4

- (d) Define a harmonic function. Give an example. $2+2=4$

(7)

8. (a) State and prove the fundamental theorem of algebra. 5

- (b) State and prove Cauchy's integral formula for a disc. $2+4=6$

- (c) Find the power series expansion of $\frac{1}{1-z}$ about $z=0$. 5

- (d) State Morera's theorem. 2
